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Hawking Radiation from NUT Kerr Newman Kusuya Black Hole via Effective Action and Covariant Anomalies

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Abstract Using the theory of the anomalous (chiral) effective action and covariant anomalies method, the Hawking radiation from NUT-Kerr-Newman-Kusuya black hole is researched. In this paper, the electric charge parameter and magnetic monopole parameter are rewritten as equivalent parameter. In addition, we simplify the metric as $1+1$ dimensional effective metric. Finally, with the method of anomalous effective action and covariant anomalies respectively, we calculate the chiral covariant current and covariant energy-momentum tensor.

Keywords Effective action · Covariant anomalies · Hawking radiation · NUT-Kerr-Newman-Kasuya black hole

1 Introduction

Since Hawking proposed the thermal radiation mechanism of black hole, the Hawking radiation of several black holes has been researched extensively [1–16]. Due to the quantum effect in curved space-time, Hawking radiation can be produced and black hole thermodynamics is founded. In black hole thermodynamics, like the thermodynamics theory, there are four basic laws, various interesting conclusions of black hole thermodynamics are derived and the conclusions could be useful to establish the theory of quantum gravity.

About the origin of Hawking radiation, one viewpoint is depicted that particles could be created and annihilated near the horizon of black holes, and negative energy particle could cross the horizon owning to the negative orbit inside of black holes, but the positive energy particle will radiate to infinity and finally form Hawking radiation. Based on the point of

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view, Robinson and Wilczek discussed the relationship between quantum anomalies theory and Hawking radiation in 2005. In their research, for the purpose of using 1 + 1 dimensional quantum anomalies equations, they successfully reduce the dimensions of space-time as 1 + 1 dimensional effective form near the horizons of black holes, and the crucial technology contributes the Hawking temperature to be obtained via anomalies theory [17–56]. Subsequently, Iso, Umetsu and Wilczek put forward new method to calculate the total flux of energy-momentum tensor and gauge flux [18–56]. Then the method of covariant anomalies is proposed, so that the calculation of anomalies theory is simplified greatly [57–62]. On the other hand, Banerjee et al. researched the anomalous (chiral) effective action, and discussed the relation between the action and Hawking radiation [63–70]. At last, the physical quantities of Hawking radiation are obtained. The method is very simple but concise, so the anomalies radiation of various black holes is studied. However, partly to blame for the difficult equation with the electric charge parameter and magnetic monopole parameter, NUT-Kerr-Newman-Kusuya black hole is still no researched, we will do the work in this letter.

2 Effective Simplified Metric of NUT-Kerr-Newman-Kusuya Black Hole

There is gravitomagnetic monopole in NUT (Newman-Unti-Tamburino) space-time, and the NUT-Kerr-Newman-Kusuya black hole is given by [71]

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - Bd\varphi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}(adt - \eta d\varphi)^2 \tag{1}$$

where

$$\begin{aligned} &\Sigma = r^2 + (n + a\cos\theta)^2 \qquad B = a\sin^2\theta - 2n\cos\theta \qquad \eta = r^2 + a^2 + n^2 \\ &\Delta = r^2 + a^2 + n^2 - 2Mr - 2n^2 + Q_h^2 + Q_m^2 \end{aligned} \tag{2}$$

Here, M, Q_h, Q_m, n and a are defined as mass, electric charge, magnetic monopole parameter, NUT parameter, and angular momentum of the black hole respectively. The event horizon r_0 of this black hole satisfies the relation $\Delta(r_0) = 0$, and the electromagnetic vector potential is

$$\hat{A} = -\frac{Q_h r}{\Sigma}(dt - Bd\varphi) + \frac{Q_m\cos\theta}{\Sigma}(adt - \eta d\varphi)^2 \tag{3}$$

It is obvious that the form of the electric charge parameter and magnetic monopole parameter lead to the trouble in the research. Therefore,we will rewrite them as equivalent charge parameter form to resolve this difficulty [72]. We consider the Maxwell equation

$$\nabla_\nu F^{\mu\nu} = 4\pi\rho_h u^\mu \qquad \nabla_\nu F^{+\mu\nu} = 4\pi\rho_m u^\mu \tag{4}$$

The $F^{\mu\nu}$ is electromagnetic tensor, and $F^{+\mu\nu}$ is its dual tensor. ρ_h, ρ_m and u^μ are the densities of electric charge, densities of magnetic charge and the 4-velocity in curved space-time. The derivative ∇_ν is covariant derivative in this space-time. For the purpose of obtaining the equivalent charge parameter and the equivalent electromagnetic tensor, we rewrite the electromagnetic tensor as

$$\hat{F}^{\mu\nu} = F^{\mu\nu}\cos\alpha + F^{+\mu\nu}\sin\alpha \tag{5}$$

where α is undetermined parameter, so the equivalent Maxwell equation is given by

$$\nabla_\nu \hat{F}^{\mu\nu} = 4\pi(\rho_h \cos\alpha + \rho_m \sin\alpha)u^\mu \qquad \nabla_\nu \hat{F}^{+\mu\nu} = 4\pi(\rho_m \cos\alpha - \rho_h \sin\alpha)u^\mu \tag{6}$$

Due to simplifying the calculation, we determine α, and make it satisfy

$$\rho_h \cos\alpha + \rho_m \sin\alpha = \rho_e \qquad \rho_m \cos\alpha - \rho_h \sin\alpha = 0 \tag{7}$$

So the equivalent charge densities is $\rho_e = \sqrt{\rho_h^2 + \rho_m^2}$, and the equivalent electromagnetic tensor is

$$A = -\frac{Q_e r}{\Sigma}(dt - Bd\varphi) \tag{8}$$

where $Q_e^2 = Q_h^2 + Q_m^2$ is equivalent charge, and therefore the metric is simplified as

$$\begin{aligned} ds^2 = &-\frac{r^2 + a^2 + n^2 - 2Mr - 2n^2 + Q_e^2}{r^2 + (n + a\cos\theta)^2}(dt - Bd\varphi)^2 + [r^2 + (n + a\cos\theta)^2]d\theta^2 \\ &+ \frac{r^2 + (n + a\cos\theta)^2}{r^2 + a^2 + n^2 - 2Mr - 2n^2 + Q_e^2}dr^2 + \frac{\sin^2\theta}{r^2 + (n + a\cos\theta)^2}(adt - \eta d\varphi)^2 \end{aligned} \tag{9}$$

The metric can describe the effect of both charge and magnetic monopole via ρ_e and A.

3 1 + 1 Dimensional Metric and Effective Action Method

However, the four dimensional metric fails to satisfy the quantum anomalies equations, so we consider to use action of the scalar field to reduce dimensions as effective 1 + 1 dimensional metric near the event horizon, and the tortoise coordinate is introduced

$$\frac{dr_*}{dr} = \frac{r^2 + (r^2 + a^2 + n^2)}{r^2 + a^2 + n^2 - 2Mr - 2n^2 + Q_e^2} \equiv f^{-1}(r) \tag{10}$$

Near the horizon r_+, Noting the nature $f \to 0$, and the wave function Φ can be rewritten as $\sum_{l,m} \phi_{l,m}(t,r)Y_{l,m}(\theta,\varphi)$, so that the spacetime can be simplified. Considering the Killing vectors $\frac{\partial}{\partial\varphi} \leftrightarrow im$ (where, m is angle momentum), at the event horizon, the simplified action of scalar field can be given by

$$S = \frac{\eta(r_0)}{2}\sum_{l,m}\int dtdr\phi_{l,m}^*\left[-f^{-1}(r)\left(\partial_t - \frac{ima + ieQ_e r}{r^2 + a^2 + n^2}\right)^2 + \partial_r(f(r)\partial_r)\right]\phi_{l,m} \tag{11}$$

where e is the equivalent charge, and the equivalent electromagnetic vector potential is

$$A_t(r) = \frac{ma + eQ_e r}{r^2 + a^2 + n^2} \tag{12}$$

Therefore, the effective 1 + 1 dimensional effective metric is

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 \tag{13}$$

and the effective metric can be used to research the anomaly radiation from black hole. It need to be pointed out that in this paper, there are two step to simplify the metric. Firstly, in (9) the equivalent charge Q_e and equivalent potential A in (8) show the total effect of both electric charge and magnetic monopole parameter; Secondly, the metric (13) and the potential (12) reflect the nature of metric (9) near the event horizon. Recently, the method of effective action is proposed to study Hawking radiation of black holes, which greatly advances the study of quantum anomaly of Hawking radiation. With this method, the effective action is introduced [73]

$$\Gamma = -\frac{1}{3} z(\omega) + z(A) \tag{14}$$

Where ω_μ and A_μ are the spin connection and equivalent gauge field, and

$$z(a) = \frac{1}{4\pi} \int dx^2 dy^2 \epsilon^{\mu\nu} \partial_\mu a_\nu(x) \nabla^{-2} \partial_\eta [(\epsilon^{\eta\rho} + g^{\eta\rho}) a_\rho(y)] \tag{15}$$

So the equivalent chiral covariant current is $J_m^\mu = m \frac{\delta\Gamma}{\delta A_\mu}$ and $J_e^\mu = e \frac{\delta\Gamma}{\delta A_\mu}$, and the energy momentum tensor is $T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta\Gamma}{\delta g^{\mu\nu}}$. Namely

$$\begin{aligned} J_e^\mu &= -\frac{eD^\mu B}{2\pi} \qquad J_m^\mu = -\frac{mD^\mu B}{2\pi} \\ T_\nu^\mu &= \frac{1}{96\pi}\left(\frac{1}{2} D^\mu H D_\nu H - D^\mu D_\nu H + \delta_\nu^\mu R\right) + \frac{1}{4\pi} D^\mu B D_\nu B \end{aligned} \tag{16}$$

where $D_\mu = \nabla_\mu - \epsilon_{\mu\nu}\nabla^\nu = -\epsilon_{\mu\nu} D^\nu$ is the chiral covariant derivative, the definition of B and H are

$$\begin{aligned} B &= \int d^2 a \nabla^{-2}(x, a) \epsilon^{\mu\nu} \partial_\mu A_\nu(a) \\ H &= \int d^2 a \nabla^{-2}(x, a) R(a) \end{aligned} \tag{17}$$

which satisfy the equations $\nabla^2 B = -\partial_r A_t(r)$ and $\nabla^2 H = R$. Solving the equations, we can obtain

$$B = B_0(r) - bt + c \qquad H = H_0(r) - 4wt + q \tag{18}$$

where $\partial_r B_0 = \frac{A_t(r)+a}{f(r)}$, $\partial_r H_0 = -\frac{f'(r)+l}{f(r)}$ and the a, b, c, w, q, l are constants. Finally, the equivalent chiral covariant current and the energy momentum tensor are given by

$$\begin{aligned} J_e^r &= \frac{e}{2\pi}[A_t(r) + a + b] \qquad J_m^r = \frac{m}{2\pi}[A_t(r) + a + b] \\ T_t^r &= \frac{1}{4\pi}[A_t(r) - A_t(r_0)]^2 + \frac{1}{192\pi}(4w - l)^2 - \frac{f'^2}{192\pi} + \frac{ff''}{96\pi} \end{aligned} \tag{19}$$

According to the fact that the equivalent chiral covariant current and the energy momentum tensor vanish at the event horizon of the black hole, we can determine the constants $a + b = -A_t(r_0)$ and $4w = \pm f'(r_0) + l$, so that the observer in infinity (where $A_t(r)$, $f'(r)$ and $f''(r)$ vanish) measures the equivalent chiral covariant current and the energy momentum tensor as

$$\begin{aligned} J_{e(0)}^r &= -\frac{eA_t(r_0)}{2\pi} \qquad J_{m(0)}^r = -\frac{mA_t(r_0)}{2\pi} \\ T_{t(0)}^r &= \frac{A_t^2(r_0)}{4\pi} + \frac{f'^2(r_0)}{192\pi} = \frac{A_t^2(r_0)}{4\pi} + \frac{\pi T_h^2}{12} \end{aligned} \tag{20}$$

where T_h is Hawking temperature of this black hole. Next section, we will study the Hawking radiation via covariant anomalies.

4 Covariant Anomalies Method and Hawking Radiation

In the two dimensional reduction, there exist both general coordinate symmetry and the gauge symmetry. If we omit the classically irrelevant ingoing modes at the event horizon, the effective theory is anomalous with respect to gauge invariance and general coordinate covariance at the quantum level. $r_0 \le r \le r_0 + \varepsilon$, the covariant anomaly satisfies

$$\nabla_\mu J^\mu_{m+} = \pm m \frac{\varepsilon^{\mu\tau}}{4\pi\sqrt{-g}} F_{\mu\tau} \qquad \nabla_\mu J^\mu_{e+} = \pm e \frac{\varepsilon^{\mu\tau}}{4\pi\sqrt{-g}} F_{\mu\tau} \tag{21}$$

where $+(-)$ are left (right)-handed field and $\varepsilon^{10} = 1$. The gauge current for the outgoing modes can be written as

$$\partial_r J^r_{m+} = \frac{m}{2\pi} \partial_r A_t \qquad \partial_r J^r_{e+} = \frac{e}{2\pi} \partial_r A_t \tag{22}$$

But in the region $r_0 + \varepsilon \le r$, the anomalies vanish, so the gauge current J^r_{eo} and J^r_{mo} is conserved

$$\partial_r J^r_{eo} = 0 \qquad \partial_r J^r_{mo} = 0 \tag{23}$$

From (22) and (23), we obtain

$$\frac{J^r_{mo}}{m} = \frac{J^r_{eo}}{e} = c_0 \tag{24}$$

$$\frac{J^r_{m+}}{m} = \frac{J^r_{e+}}{e} = c_+ + \frac{1}{2\pi} \left(A_t(r) - A_t(r_+) \right) \tag{25}$$

where c_o and c_+ are the gauge current flux at the infinity and the horizon, respectively. The total gauge current therefore can be written as

$$J^r_m = J^r_{mo} \Theta_+(r) + J^r_{m+} H(r) \qquad J^r_e = J^r_{eo} \Theta_+(r) + J^r_{e+} H(r) \tag{26}$$

where

$$\Theta_+(r) = \Theta(r - r_0 - \varepsilon) \qquad H(r) = 1 - \Theta_+(r) \tag{27}$$

and

$$\Theta(r - r_0 - \varepsilon) = \begin{cases} 1 & r_0 + \varepsilon < r \\ 0 & r_0 < r < r_0 + \varepsilon \end{cases} \tag{28}$$

Using the conservation equation, we can obtain

$$\begin{aligned} \partial_\mu J^\mu_m &= \partial_r J^r_m = \left(J^r_{mo} - J^r_{m+} + \frac{m A_t}{2\pi} \right) \delta(r - r_0 - \varepsilon) + \partial_r \left(\frac{m A_t}{2\pi} H \right) \\ \partial_\mu J^\mu_e &= \partial_r J^r_e = \left(J^r_{eo} - J^r_{e+} + \frac{e A_t}{2\pi} \right) \delta(r - r_0 - \varepsilon) + \partial_r \left(\frac{e A_t}{2\pi} H \right) \end{aligned} \tag{29}$$

where the last term is the boundary term from quantum effect, and therefore should be omitted, then we get

$$J^r_{mo} = J^r_{m+} - \frac{mA_t(r)}{2\pi} \qquad J^r_{eo} = J^r_{e+} - \frac{eA_t(r)}{2\pi} \tag{30}$$

we therefore have

$$c_o = c_+ - \frac{A_t(r_0)}{2\pi} \tag{31}$$

As the covariant current is vanished at the event horizon, so we obtain

$$c_+ = 0 \tag{32}$$

Now, the gauge current flux required to cancel covariant gauge anomaly at the event horizon is

$$c_o = -\frac{A_t(r_0)}{2\pi} \tag{33}$$

Next, we research the compensating energy-momentum tensor flux. As the Ward identical equation could be written as

$$\nabla_\mu T^\mu_\tau = F_{\mu\tau} J^\mu + A_\tau \tag{34}$$

where A_τ need to satisfies

$$\nabla_\mu T^\mu_\tau = \frac{\varepsilon^{\mu\tau}}{96\pi\sqrt{-g}} \partial^\tau R = A_\tau \tag{35}$$

where

$$\begin{aligned} A_t &= \partial_\tau N^r_t \\ N^r_t &= \frac{(-f'^2 + 2ff'')}{192\pi} \end{aligned} \tag{36}$$

In $r_0 + \varepsilon \le r r_+ + \varepsilon \le r$, without any anomaly, the covariant energy momentum tensor is

$$\partial_r T^r_{t(o)} = F_{rt} \tilde{J}^r_o \tag{37}$$

In other region $r_0 \le r \le r_0 + \varepsilon$, the covariant energy momentum tensor should satisfy the anomalous equation

$$\partial_r T^r_{t(+)} = F_{rt} J^r_+ + \partial_r N^r_t \tag{38}$$

The total covariant energy momentum tensor can be given by

$$T^\mu_\upsilon = T^\mu_{\upsilon(o)} \Theta_+(r) + T^\mu_{\upsilon(+)} H(r) \tag{39}$$

Then we have

$$\begin{aligned} \nabla_\mu T^\mu_t &= \partial_r T^r_t \\ &= \left(T^r_{t(o)} - T^r_{t(+)} + N^r_t + \frac{A_t^2}{4\pi} \right) \delta(r - r_+ - \varepsilon) \\ &\quad + \partial_r \left[\left(N^r_t + \frac{A_t^2}{4\pi} \right) H \right] + c_o \partial_r A_t(r) \end{aligned} \tag{40}$$

where the second and the third term are quantum boundary effect and the classical effect of the background electromagnetic field for current flow, so

$$T^r_{t(o)} = T^r_{t(+)} - N^r_t + \frac{A_t^2}{4\pi} \tag{41}$$

At the event horizon, we have

$$k_o = k_+ + \frac{A_t^2(r_0)}{4\pi} - N_t^r(r_+) \tag{42}$$

where

$$k_o = T^r_{t(o)} - c_o A_t(r_0) \tag{43}$$

Imposing the vanishing of the covariant energy-momentum tensor current at the horizon, namely

$$k_+ = T^r_{t(+)} - \int_{r_+}^{r} dr\,\partial_r \left[c_o A_t(r_0) + \frac{A_t^2}{4\pi} + N_t^r\right] = 0 \tag{44}$$

So the compensating energy-momentum tensor flux can be finally expressed as

$$k_{(o)} = \frac{A_t^2(r_0)}{4\pi} + \frac{\pi}{12} T_h^2 \tag{45}$$

where $T_h = \partial_r f(r)/(4\pi)|_{r=r_0}$ is the temperature of this black hole.

5 Conclusion

For fermions, as we all know, the Hawking radiation spectrum from the horizon of the black hole is

$$N_\pm(\omega) = 1\Big/\left[\exp\left(\frac{\omega \pm A_t(r_0)}{T_h}\right) + 1\right] \tag{46}$$

so the angular momentum current and the energy-momentum tensor fluxes can be, respectively, given by

$$F_J = \frac{F_{Jm}}{m} = \frac{F_{em}}{e} = \frac{1}{2\pi}\int_0^\infty \left[N_+(\omega) - N_-(\omega)\right] d\omega = -\frac{A_t(r_0)}{2\pi} \tag{47}$$

and

$$F_T = \frac{1}{2\pi}\int_0^\infty \omega\left[N_+(\omega) + N_-(\omega)\right] d\omega = \frac{A_t^2(r_0)}{4\pi} + \frac{\pi}{12} T_h^2 \tag{48}$$

From the results in (20), (31), (45) and (47), (48), we can find that our results are right and reasonable.

In covariant anomalies and effective action method, several conditions depend on the vacuum state, and some results can be gotten from the physical quantity [18–56, 63–69], so the researches about Hawking radiation can help us to understand the essence of vacuum. It is obvious, in this paper, that the results at the event horizon are reasonable and the method simplifies the research greatly. When $Q_m = 0$, the results change to be the same situation of NUT-Kerr-Newman space-time, and the conclusions are the results of Kerr-Newman black hole, when $n = Q_m = 0$. In this letter, we used equivalent Einstein-Maxwell theory to rewrite the metric, and then the metric is reduced dimensions as $1+1$ dimensional effective metric. In the end, the equivalent chiral covariant current and the energy momentum tensor are obtained. The conclusion shows that the Hawking temperature at the event horizon is $T_h = \frac{f'(r_0)}{4\pi}$, which accords with the result of tunneling method and other methods. Since many famous black holes are special cases of NUT-Kerr-Newman-Kusuya black hole, our research in this letter is significative. However, as we all know, the real case in astronomy should be non-stationary black hole, whose metric should depend on time, so the research about non-stationary black hole will be more important. It is no other than our further work.

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